

Pseudoscalar Form Factors in Tau-Neutrino Nucleon Scattering

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Abstract

We investigate the pseudoscalar transition form factors of nucleon for quasi-elastic scattering and Δ resonance production in tau-neutrino nucleon scattering via the charged current interactions. Although the pseudoscalar form factors play an important role for the τ production in neutrino-nucleon scattering, these are not known well. In this article, we examine their effects in quasi-elastic scattering and Δ resonance production and find that the cross section, Q^2 distribution, and spin polarization of the produced τ^\pm leptons are quite sensitive to the pseudoscalar form factors.

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Neutrino oscillations in long baseline (LBL) experiments are of great interests from both theoretical and experimental point of view. It is especially important to confirm $\nu_\mu \rightarrow \nu_\tau$ oscillation, and ν_τ should be detected through the τ production by charged current reactions off a nucleon target at several LBL neutrino oscillation experiments [1], such as MINOS [2], ICARUS [3] and OPERA [4].

As we pointed out in the previous paper [5], the information on the spin polarization of τ produced through the neutrino-nucleon scattering is essential to determine the τ^\pm production signal since the decay particle distributions depend crucially on the τ polarization. τ production followed by its pure leptonic decay should also be studied in order to estimate background events for the $\nu_\mu \rightarrow \nu_e$ appearance reactions [6]. Furthermore, in addition to LBL experiments, the ice/underwater neutrino telescopes are expected to detect τ production in neutrino-nucleon scattering, such as at AMANDA [7]/IceCube [8] and Baikal [9] experiments.

In LBL experiments, the following three reactions have major contribution to the neutrino-nucleon scattering; quasi-elastic scattering (QE), resonance production (RES), and deep inelastic scattering (DIS). The QE contribution to the τ production dominates the total cross section near threshold, $E_\nu \sim 3.5$ GeV, and the cross sections of the QE and RES processes are significant throughout the energy range of $E_\nu \simeq 3.5 - 30$ GeV of the future neutrino oscillation experiments [10, 5]. It is thus important to estimate the τ production cross section and its spin polarization for the QE and RES processes.

However, there is an uncertainty in the calculations of the cross section and spin polarization of τ production, from the pseudoscalar form factors of those processes. Because the contribution from the pseudoscalar form factors is proportional to the lepton mass, for the e and μ production case these contributions are suppressed and negligible. Although there are several experiments which have sensitivity to the pseudoscalar form factors, such as in muon capture [11] and in pion electroproduction [12], those results are not sufficient to constrain these form factors in the range relevant for τ production [13]. On the other hand, because of the heavy τ mass, $m_\tau = 1.78$ GeV, the effect of pseudoscalar terms to the τ production can be significant since their spin-flip nature is expected to affect the produced τ polarization significantly.

In this letter, we study τ production in the neutrino-nucleon scattering using several parameterizations of the pseudoscalar form factors in the QE and RES processes, and examine how the production cross section and the spin polarization of τ are affected by those form factors.

We consider τ^-/τ^+ production by charged current reactions off a nucleon target;

$$\nu_\tau(k)/\bar{\nu}_\tau(k) + N(p) \rightarrow \tau^-(k')/\tau^+(k') + X(p'), \quad (1)$$

where the four-momenta are given in brackets and X denotes the final hadron. X is a nucleon N for the QE process and Δ or $N + \pi$ for the RES process. We define Lorentz invariant variables

$$Q^2 = -q^2, \quad q = k - k', \quad (2)$$

$$W^2 = (p + q)^2, \quad (3)$$

where Q^2 is the momentum transfer. Each process is distinguished by the hadronic invariant mass W : $W = M$ for QE, and $M + m_\pi < W < W_{\text{cut}}$ for RES. Here W_{cut} is an artificial boundary between the RES and DIS ($W > W_{\text{cut}}$) processes, and we take $W_{\text{cut}} = 1.6$ GeV [5].

The τ production cross section is expressed in terms of the leptonic tensor $L^{\mu\nu}$ and the hadronic tensor $W_{\mu\nu}$ as

$$\frac{d\sigma_\lambda}{dQ^2 dW^2} = \frac{G_F^2 \kappa^2}{16\pi M^2 E_\nu^2} L_\lambda^{\mu\nu} W_{\mu\nu}, \quad (4)$$

where G_F is Fermi constant, $\kappa = M_W^2/(Q^2 + M_W^2)$ is the propagator factor with the W -boson mass M_W , M is the nucleon mass, and E_ν is the incoming τ neutrino energy in the laboratory frame. λ stands for the produced τ helicity defined in the center-of-mass (CM) frame. Explicit form of the

leptonic tensor $L_{\lambda}^{\mu\nu}$ in terms of the $\nu_{\tau} \rightarrow \tau_{\lambda}^{-}$ and $\bar{\nu}_{\tau} \rightarrow \tau_{\lambda}^{+}$ transition currents, for the τ^{\pm} helicity λ in the CM frame, is found in Ref. [5].

The hadron tensor for the QE scattering processes

$$\nu_{\tau} + n \rightarrow \tau^{-} + p, \quad \bar{\nu}_{\tau} + p \rightarrow \tau^{+} + n, \quad (5)$$

is written by using the hadronic weak transition current $J_{\mu}^{(\pm)}$ as follows [14]:

$$W_{\mu\nu}^{\text{QE}} = \frac{\cos^2 \theta_c}{4} \sum_{\text{spins}} J_{\mu}^{(\pm)} J_{\nu}^{(\pm)*} \delta(W^2 - M^2), \quad (6)$$

where θ_c is the Cabibbo angle. The weak transition currents $J_{\mu}^{(+)}$ and $J_{\mu}^{(-)}$ for the ν_{τ} and $\bar{\nu}_{\tau}$ scattering, respectively, are defined as

$$J_{\mu}^{(+)} = \langle p(p') | \hat{J}_{\mu}^{(+)} | n(p) \rangle = \bar{u}_p(p') \Gamma_{\mu}(p', p) u_n(p), \quad (7)$$

$$J_{\mu}^{(-)} = \langle n(p') | \hat{J}_{\mu}^{(-)} | p(p) \rangle = \bar{u}_n(p') \bar{\Gamma}_{\mu}(p', p) u_p(p) = \langle p(p) | \hat{J}_{\mu}^{(+)} | n(p') \rangle^*, \quad (8)$$

where Γ_{μ} is written in terms of the six weak form factors of the nucleon, $F_{1,2,3}^V$, F_A , F_3^A and F_p , as

$$\begin{aligned} \Gamma_{\mu}(p', p) = & \gamma_{\mu} F_1^V(q^2) + \frac{i\sigma_{\mu\alpha} q^{\alpha} \xi}{2M} F_2^V(q^2) + \frac{q_{\mu}}{M} F_3^V(q^2) \\ & + \left[\gamma_{\mu} F_A(q^2) + \frac{(p+p')_{\mu}}{M} F_3^A(q^2) + \frac{q_{\mu}}{M} F_p(q^2) \right] \gamma_5. \end{aligned} \quad (9)$$

For the $\bar{\nu}_{\tau}$ scattering, the vertex $\bar{\Gamma}_{\mu}$ is obtained by $\bar{\Gamma}_{\mu}(p', p) = \gamma_0 \Gamma_{\mu}^{\dagger}(p, p') \gamma_0$.

We can drop two form factors, F_3^V and F_3^A , because of isospin symmetry and time reversal invariance. Moreover, the vector form factor F_1^V and F_2^V are related to the electromagnetic form factors of nucleons under the conserved vector current (CVC) hypothesis:

$$F_1^V(q^2) = \frac{G_E^V(q^2) - \frac{q^2}{4M^2} G_M^V(q^2)}{1 - \frac{q^2}{4M^2}}, \quad \xi F_2^V(q^2) = \frac{G_M^V(q^2) - G_E^V(q^2)}{1 - \frac{q^2}{4M^2}}, \quad (10)$$

where

$$G_E^V(q^2) = \frac{1}{(1 - q^2/M_V^2)^2}, \quad G_M^V(q^2) = \frac{1 + \xi}{(1 - q^2/M_V^2)^2}, \quad (11)$$

with a vector mass $M_V = 0.84$ GeV and $\xi = \mu_p - \mu_n = 3.706$. μ_p and μ_n are the anomalous magnetic moments of proton and neutron, respectively. For the axial vector form factor F_A ,

$$F_A(q^2) = \frac{F_A(0)}{(1 - q^2/M_A^2)^2} \quad (12)$$

with $F_A(0) = -1.267$ and an axial vector mass $M_A = 1.0$ GeV. The above form factors are found to reproduce the ν_{μ} and $\bar{\nu}_{\mu}$ scattering data [13].

For the pseudoscalar form factor F_p , which is the main focus of this study, we adopt the following parameterizations with different powers of $(1 - q^2/M_A^2)$:

$$F_p(q^2) = \frac{2M^2}{m_{\pi}^2 - q^2} \frac{F_A(0)}{(1 - q^2/M_A^2)^n} \quad (n = 0, 1, 2). \quad (13)$$

The normalization of $F_p(0)$ is fixed by the partially conserved axial vector current (PCAC) hypothesis. We adopted $n = 2$ in the previous study [5].

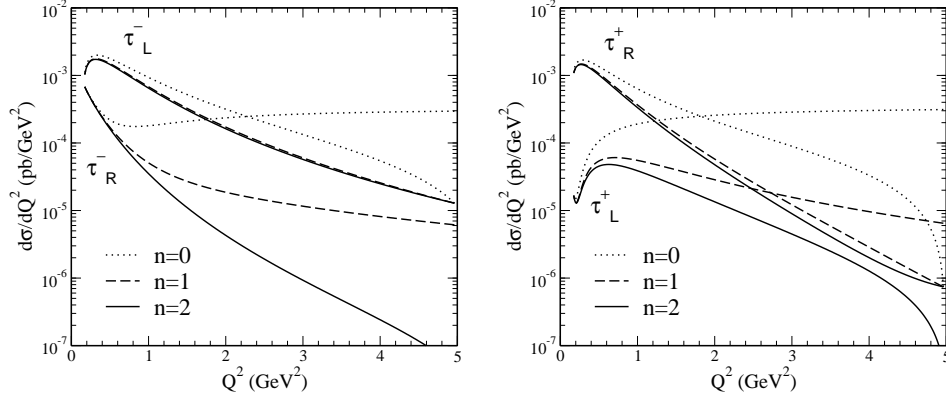


Fig. 1: Differential cross sections $d\sigma/dQ^2$ of τ^- (left) and τ^+ (right) productions off the isoscalar target in the QE process at neutrino energy $E_\nu = 5$ GeV. The right- and left-handed τ production are shown separately, where the helicities are defined in the CM frame. Solid, dashed, and dotted lines denote $n = 2, 1, 0$, respectively, for the pseudoscalar form factor Eq. (13).

In Fig. 1, we plot the $Q^2 (= -q^2)$ dependence of the differential cross sections of τ production off the isoscalar target in the QE process at incoming neutrino energy $E_\nu = 5$ GeV in the laboratory frame. The left figure is for τ^- production and the right figure is for τ^+ production. In the left figure, the upper three lines are for the left-handed τ^- (τ_L^-) production, and the lower three are for the right-handed τ^- (τ_R^-) production. Here the helicity is defined in the CM frame. On the other hand, in the right figure, upper three lines denote right-handed τ^+ (τ_R^+) and lower three for left-handed τ^+ (τ_L^+). Solid, dashed, and dotted lines are for $n = 2, 1, 0$, respectively.

We find, while the left-handed τ^- and the right-handed τ^+ production do not depend much on the pseudoscalar form factor, the dependences of the right-handed τ^- and the left-handed τ^+ production on the power of $(1 - q^2/M_\Delta^2)$ of the pseudoscalar form factor are quite significant, especially at large Q^2 . This feature agrees with the spin-flip nature of the pseudoscalar form factor. The $n = 0$ lines (the pion-pole dominance) give a characteristic prediction that the cross sections for spin-flipped τ 's (τ_R^- and τ_L^+) grow at high Q^2 . Therefore it should be possible to distinguish between the $n = 0$ and $n \geq 1$ cases. On the other hand, the difference between the $n = 1$ and the $n = 2$ cases is rather hard to be established since the cross section is very small in the large Q^2 region where the difference becomes large.

Next, the hadron tensor for the Δ production (RES) processes;

$$\nu_\tau + n(p) \rightarrow \tau^- + \Delta^+ (\Delta^{++}), \quad \bar{\nu}_\tau + p(n) \rightarrow \tau^+ + \Delta^0 (\Delta^-), \quad (14)$$

is calculated in terms of the nucleon- Δ weak transition current J_μ as follows [14, 15, 16]:

$$W_{\mu\nu}^{\text{RES}} = \frac{\cos^2 \theta_c}{4} \sum_{\text{spins}} J_\mu J_\nu^* \frac{1}{\pi} \frac{W\Gamma(W)}{(W^2 - M_\Delta^2)^2 + W^2\Gamma^2(W)}. \quad (15)$$

Here we take the Δ resonance mass $M_\Delta = 1.232$ GeV, and its running width:

$$\Gamma(W) = \Gamma(M_\Delta) \frac{M_\Delta}{W} \frac{\lambda^{\frac{1}{2}}(W^2, M^2, m_\pi^2)}{\lambda^{\frac{1}{2}}(M_\Delta^2, M^2, m_\pi^2)} \quad (16)$$

with $\Gamma(M_\Delta) = 0.12$ GeV and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$.

The current J_μ for the process $\nu_\tau + n \rightarrow \tau^- + \Delta^+$ is defined by

$$J_\mu = \langle \Delta^+(p') | \hat{J}_\mu | n(p) \rangle = \bar{\psi}^\alpha(p') \Gamma_{\mu\alpha} u(p), \quad (17)$$

where ψ^α is the spin-3/2 particle wave function and the vertex $\Gamma_{\mu\alpha}$ is expressed in terms of the eight weak form factors $C_{i=3,4,5,6}^{V,A}$ as

$$\begin{aligned}\Gamma_{\mu\alpha} = & \left[\frac{g_{\mu\alpha} \not{q} - \gamma_\mu q_\alpha}{M} C_3^V(q^2) + \frac{g_{\mu\alpha} p' \cdot q - p'_\mu q_\alpha}{M^2} C_4^V(q^2) \right. \\ & \left. + \frac{g_{\mu\alpha} p \cdot q - p_\mu q_\alpha}{M^2} C_5^V(q^2) + \frac{q_\mu q_\alpha}{M^2} C_6^V(q^2) \right] \gamma_5 \\ & + \frac{g_{\mu\alpha} \not{q} - \gamma_\mu q_\alpha}{M} C_3^A(q^2) + \frac{g_{\mu\alpha} p' \cdot q - p'_\mu q_\alpha}{M^2} C_4^A(q^2) \\ & + g_{\mu\alpha} C_5^A(q^2) + \frac{q_\mu q_\alpha}{M^2} C_6^A(q^2).\end{aligned}\quad (18)$$

By isospin invariance and the Wigner-Eckart theorem, the other nucleon- Δ weak transition currents are given as

$$\langle \Delta^{++} | \hat{J}_\mu | p \rangle = \sqrt{3} \langle \Delta^+ | \hat{J}_\mu | n \rangle = \sqrt{3} \langle \Delta^0 | \hat{J}_\mu | p \rangle = \langle \Delta^- | \hat{J}_\mu | n \rangle. \quad (19)$$

From the CVC hypothesis, $C_6^V = 0$ and the other vector form factors $C_{i=3,4,5}^V$ are related to the electromagnetic form factors. We adopt the modified dipole parameterizations [17, 18]:

$$C_3^V(q^2) = \frac{C_3^V(0)}{\left(1 - \frac{q^2}{M_V^2}\right)^2} \frac{1}{1 - \frac{q^2}{4M_V^2}}, \quad C_4^V(q^2) = -\frac{M}{M_\Delta} C_3^V(q^2), \quad C_5^V(q^2) = 0, \quad (20)$$

with $C_3^V(0) = 2.05$ and a vector mass $M_V = 0.735$ GeV.

For the axial vector form factors $C_{i=3,4,5}^A$, several theoretical works were done around 1960–70 [19]–[23]. Several authors [14, 15] performed the comparisons of these predictions in detail with experimental data, and showed that the Adler model [21] modified by Schreiner and von Hippel [15] describes the data well at the time. However, in face of the recent experimental data [24] the Q^2 dependence of the weak axial form factors has been re-examined, and several authors proposed modified weak axial form factors [26, 17]. We show the several models for C_5^A as examples:

$$C_5^A(q^2) = \frac{C_5^A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2} \times \begin{cases} 1 & \text{dipole model} \\ \left(1 - \frac{a_A q^2}{b_A - q^2}\right) & \text{modified Adler model [15]} \\ \exp\left[-\frac{a_B q^2}{1 - b_B q^2}\right] & \text{Bell et al. model [25]} \\ (1 - a_S q^2) \exp[b_S q^2] & \text{SL model [26]} \\ \frac{1}{1 - \frac{q^2}{3M_A^2}} & \text{PYS model [17]} \end{cases} \quad (21)$$

with $C_5^A(0) = 1.2$, an axial vector mass $M_A = 1.0$ GeV. $a_{A,B,S}$ and $b_{A,B,S}$ are the model dependent parameters determined by fitting the experimental data, $a_{A,B,S} = -1.21, -0.61, 0.154$ and $b_{A,B,S} = 2.0, 0.19, 0.166$, when q^2 is measured in units of GeV^2 . For C_3^A and C_4^A , $C_3^A = 0$ and $C_4^A = -\frac{1}{4}C_5^A$ give good agreements with the data [15]. In this report, we adopt the PYS model [17] in Eq. (21), which decrease more rapidly with increasing Q^2 than the dipole model and the SL model [26], and which have more moderate Q^2 dependence as compared to the modified Adler model [15] and the Bell et al. model [25].

For the pseudoscalar form factor C_6^A , we adopt the same form of the parameterizations as for the QE case:

$$C_6^A(q^2) = \frac{M^2}{m_\pi^2 - q^2} \frac{C_5^A(0)}{(1 - q^2/M_A^2)^n} \quad (n = 0, 1, 2) \quad (22)$$

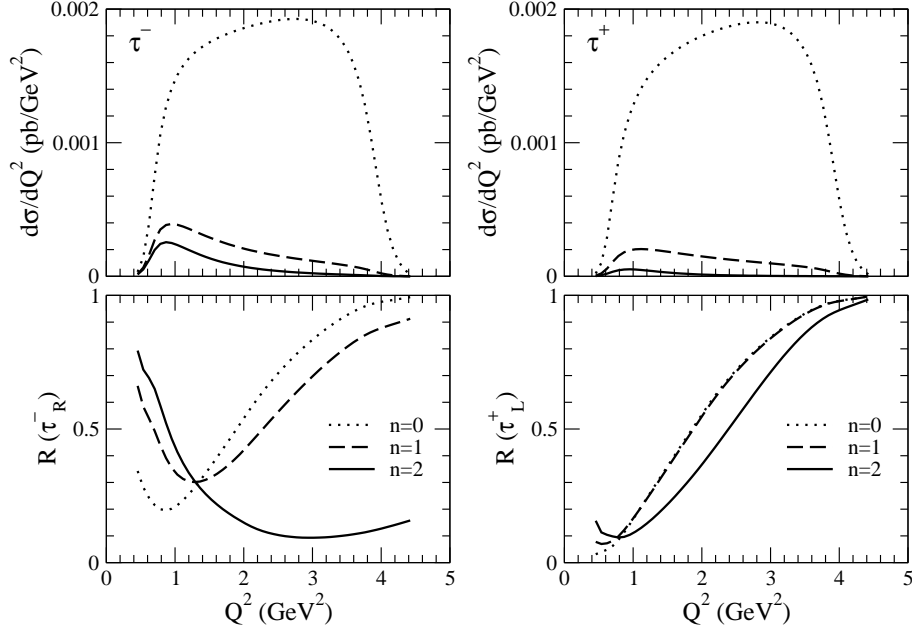


Fig. 2: Q^2 dependence of the differential cross section (upper) and the ratio of the spin-flipped τ production cross section (lower) defined in Eq. (23) for τ^- (left) and τ^+ (right) productions off the isoscalar target in the RES process at neutrino energy $E_\nu = 5$ GeV, where the helicities are defined in the CM frame. Solid, dashed, and dotted lines denote $n = 2, 1, 0$, respectively, for the pseudoscalar form factor Eq. (22).

which agrees with the off-diagonal Goldberger-Treiman relation in the limit of $m_\pi^2 \rightarrow 0$ and $q^2 \rightarrow 0$.

In Fig. 2, we show the cross section and polarization of produced τ separately. The upper two figures show the Q^2 dependence of the differential cross section for τ^- (left figure) and τ^+ (right figure) production off the isoscalar target in the RES process at neutrino energy $E_\nu = 5$ GeV. Solid, dashed, and dotted lines as for $n = 2, 1, 0$, respectively. Both τ^- and τ^+ production cross sections for $n = 0$ are almost 10 times larger than those for $n = 1, 2$. This is because of the absence of extra Q^2 suppression to the pion-pole term in the pseudoscalar form factor. Unlike the case for the QE process, the Δ production cross sections can distinguish between $n = 1$ and $n = 2$ cases.

The lower figures show the ratio of the cross section of spin-flipped τ production, τ_R^- (left figure) and τ_L^+ (right figure), defined as

$$R(\tau_R^-) = \frac{d\sigma_R}{dQ^2} \bigg/ \frac{d\sigma}{dQ^2}, \quad R(\tau_L^+) = \frac{d\sigma_L}{dQ^2} \bigg/ \frac{d\sigma}{dQ^2}, \quad (23)$$

where $d\sigma = d\sigma_R + d\sigma_L$. Here the helicity is defined in the CM frame, as above. The helicity ratios shown in the lower figures give qualitatively different results between τ^- and τ^+ productions. For τ^- , in the large Q^2 region, the left-handed τ^- dominates for $n = 2$, while the right-handed τ^- dominates for $n = 0, 1$. On the other hand, for τ^+ , the large Q^2 region is dominated by left-handed τ^+ for all $n = 0, 1, 2$ cases. Only left-handed τ^+ are produced in the backward direction in the CM frame.

We also examined the parametrization

$$C_6^A(q^2) = \frac{M^2}{m_\pi^2 - q^2} \cdot C_5^A(q^2) \quad (24)$$

by using the PYS parametrization [17] of the weak axial vector form factor in Eq. (21). We find negligible difference from the $n = 2$ case, for the cross section prediction and the polarization prediction

in the region where the cross section is relatively significant.

To summarize, we have studied the pseudoscalar transition form factors of nucleon for quasi-elastic scattering and Δ resonance production in tau-neutrino nucleon scattering via the charged current interactions. Q^2 dependence of the τ^\pm cross sections was calculated, considering the helicities of τ defined in the CM frame. We found that the pseudoscalar form factors significantly enhance spin-flip τ production, right-handed τ^- and left-handed τ^+ production, and that it is possible to determine whether those form factors need extra Q^2 suppression to the pion-pole term or not. Δ resonance cross sections are sensitive to the degree of extra Q^2 suppression.

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